PHYSCIWKSHT. Range of Projectile

After you have split the initial velocity of a projectile into its **vertical** and **horizontal** components, you can use Newton's equations of motion under the influence of **constant acceleration** to solve for the time (in seconds) of the projectile's flight. The two great advantages of studying motion under the influence of gravity are

1. The acceleration of gravity **g** (-9.8 m/s^2) is nearly constant near the Earth's surface. It is negative since gravity points downward

2. Gravity points downward (the negative **j** direction)
3. There is no acceleration in the **i** or horizontal direction if air resistance is ignored

Newton's Equations of Motion

$$v_{final}^2 = v_{initial}^2 + 2ad$$

$$v_{final} = v_{initial} + at$$

$$v_{average} = \frac{distance}{time}$$

$$v_{average} = \frac{\left(v_{initial} + v_{finall}\right)}{2}$$

$$v_{final} = v_{initial} + a \cdot t$$

$R = R_{initial} + V_{initial} \cos\theta \cdot t + \frac{1}{2}at^2$ (in the horizontal direction a=0)

so this reduces to

$$R = R_{initial} + V_{initial} \cos\theta \cdot t$$

Where R is the Range or horizontal distance the projectile travels

In the vertical direction

$h = h_{initial} + V_{initial} \sin\theta \cdot t + \frac{1}{2}at^2$ Since in the vertical direction a=$\boldsymbol{g}$ = -9.8 m/s^2

this reduces to

$h = h_{initial} + V_{initial} \sin\theta \cdot t - \frac{1}{2}gt^2$ (g is negative since gravity points down)

In all the above equations,

v or V stands for velocity and is measured in meters/second

t stands for time and is measured in seconds

a stands for acceleration and is measured in m/s^2

h stands for height and is measured in meters

To solve projectile motion problem, the key is to

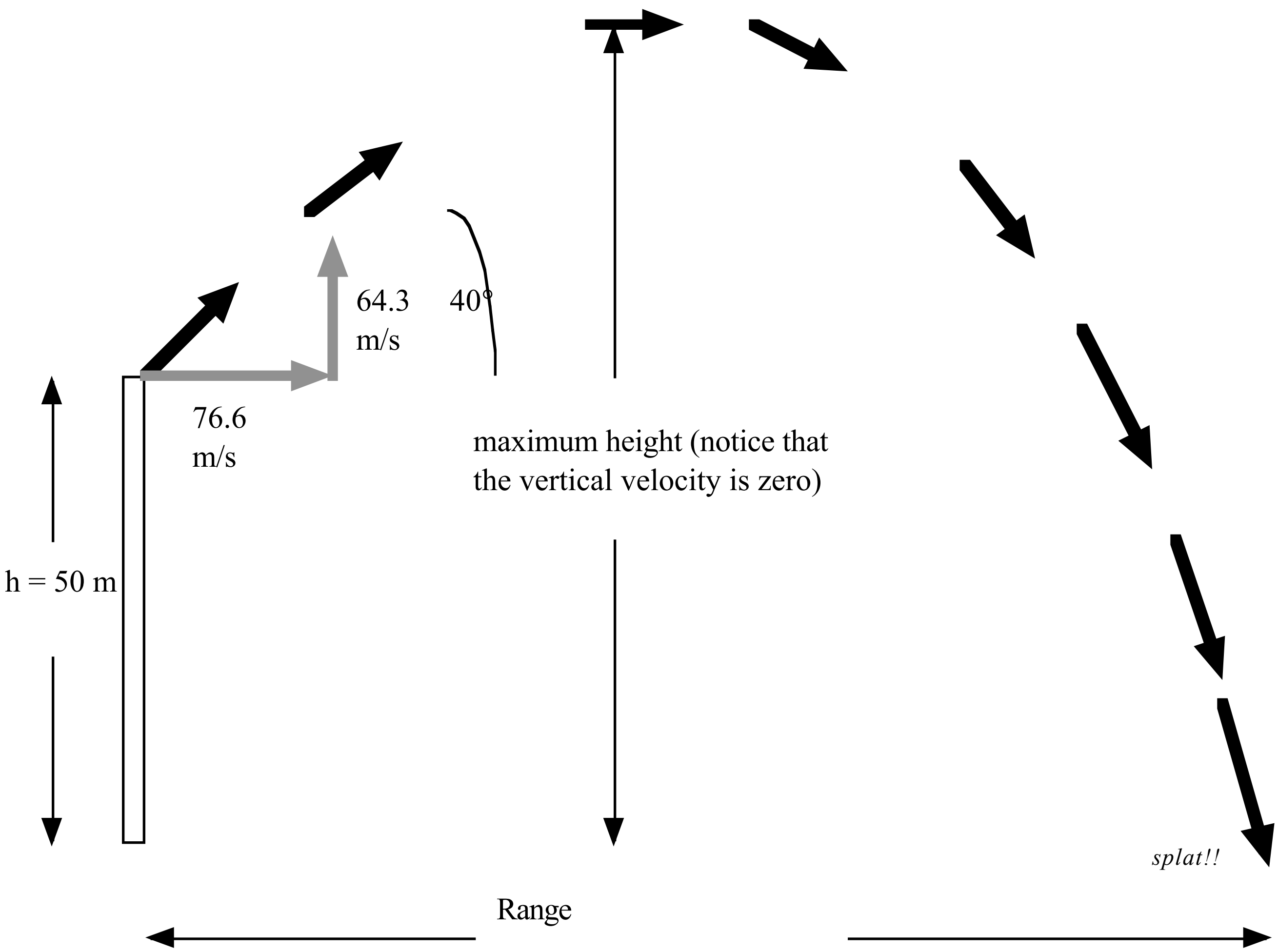
- separate the motion into vertical and horizontal components
- solve for the time (t) of the motion. *You will usually use the vertical motion to do this part*

This time is what connects the horizontal and vertical components together.

- Find the range in meters using Newton's equations
- Find the maximum height that the projectile reaches
- Find the final vertical velocity using Newton's equations
- The horizontal velocity does not change. You can get the final *net* velocity (magnitude and direction) from the final horizontal (**i**) and final vertical (**j**) velocities

Example:

A projectile is launched off the edge of a cliff with an initial velocity of 100 m/s at an angle of 40.0° above horizontal. The cliff is 50 meters high.



$h_{initial}$ = 50 m $V_{horizontal}$ = 100m/s•cos 40 = 76.6 m/s**i**

h_{final} = 0 m $V_{vertical}$ = 100m/s•sin40 = 64.3 m/s

t = ??   Range (horizontal distance traveled)= ????

$V_{final(vertical)}$ = ??? Maximum height = ??????

$V_{final(horizontal)}$= ????

A. *How long does it take (in seconds) for the projectile to reach the ground?*

Since $h = h_{initial} + V_{initial}\sin\theta \cdot t - \frac{1}{2}gt^2$

We get $0 = 50 + 64.3 \cdot t - \frac{1}{2}(9.8)t^2$

This is a quadratic equation where

a = -4.9   b= 64.3   c = 50

Solving gives us

$$\frac{-64.3 \pm \sqrt{64.3^2 - 4 \cdot (-4.9) \cdot 50}}{2(-4.9)}$$

t = -0.736 s and t = 13.85 s

Negative time makes no sense so we choose the positive answer. Thus it takes 13.8 seconds for the projectile to reach the ground.

B. *How far does it land from the bottom of the cliff (what is its range)?*

$R = R_{initial} + V_{initial}\cos\theta \cdot t$

Since $R_{initial}$ *= 0 meters (where it started from)*

R= 76.6 m/s•13.8 s= 1057 meters (1.06 x 10^3 *m)*

C. *What is the maximum height that the projectile reaches?*

Since $v^2 = v_{initial}^2 + 2ad$

We can solve this in the <u>vertical direction only</u>

<u>We can use the fact that at its maximum height the vertical velocity = 0 m/s</u>

$0 = v_{initialvertical}^2 + 2(-9.8)h$

$$h = \frac{v_{initialvertical}^2}{2 \cdot (9.8)}$$

$$h = \frac{64.3^2}{19.6} = 211m$$

to this add the initial height of 50 meters. Therefore, the projectile goes to a height of 261 m

D. *What is the final vertical velocity of the projectile?*

$$v_{final} = v_{initial} + a \cdot t$$
$$v_{final(vertical)} = v_{initial(vertical)} - 9.8 \cdot t$$
$$v_{final(vertical)} = 64.3 - 9.8 \cdot (13.8)$$
$$v_{final(vertical)} = -71.0 m / s$$

*This is negative because the projectile is headed down (**j** is negative)*

E. *What is the final horizontal velocity of the projectile*

Since there is no acceleration in the horizontal direction the final horizontal velocity is the same as the initial horizontal velocity, or
76.6 m/s **i**

F. *What is the magnitude of the final net velocity?*

$$V_{final} = (76.6i - 71j)\, m / s$$
$$V_{final} = \sqrt{(76.6^2) + (-71.0^2)}$$
$$V_{final} = 105 m / s$$

G. *What is the direction of the final net velocity?*

$$\theta = \tan^{-1}\left(\frac{j}{i}\right)$$
$$\theta = \tan^{-1}\left(\frac{-71.0}{76.6}\right)$$
$$\theta = -42.8°$$

This means that the direction of the final velocity is 42.8° below horizontal (down and to the right)